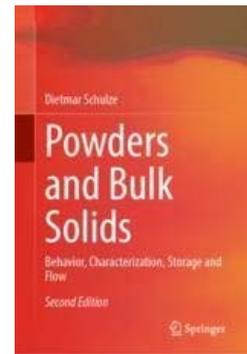


More information on flow properties and handling of powders and bulk solids can be found in:

**Powders and Bulk Solids**  
– Behavior, Characterization, Storage and Flow

by Dietmar Schulze

2nd ed. 2021, published by Springer  
(Link: <https://www.springer.com/us/book/9783030767198>)



## Storage of Powders and Bulk Solids in Silos

Dietmar Schulze<sup>1</sup>

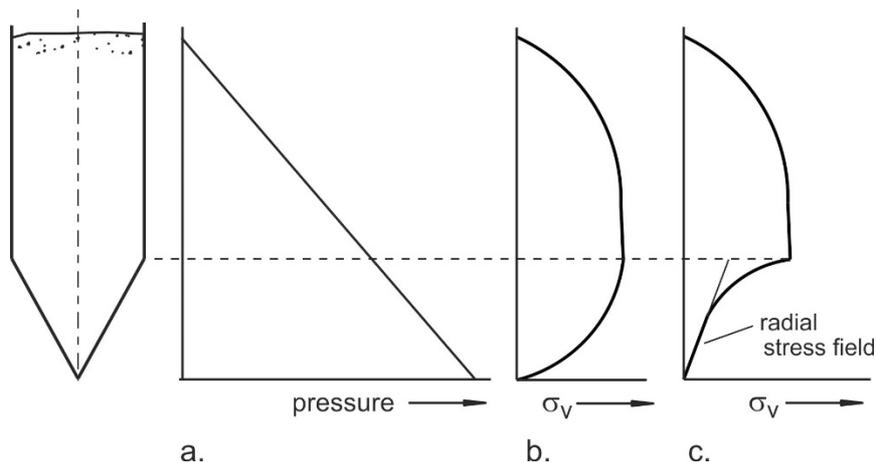
*When storing powders or bulk materials in improperly designed silos, problems often arise which could be avoided by designing the silos according to bulk solid's flow properties (a procedure that is the rule for other types of equipment, e.g. heat exchangers). After a brief description of the problems that can arise in poorly designed silos, this essay deals with the basic rules for designing silos.*

### 1 Stresses in silos

Figure 1 shows silos with the associated pressure and stress distributions. While one usually speaks of pressure in the case of liquids, the “bulk solid pressure” is referred to below as stress. If a silo were filled with a liquid, the pressure would increase linearly downwards regardless of the silo cross-section and the inclination of the walls.

In contrast, in a silo filled with bulk solid, the stress distribution is different: In the silo's vertical section, the vertical stress increases less and less downwards, until finally (with a sufficiently large height / diameter ratio, usually greater than 3) a nearly constant vertical stress is achieved. This means that the vertical stress in the vertical section would not increase any further, even at much higher filling levels. The cause of this stress distribution are the shear stresses exerted by the bulk solid on the silo wall, which carry part of the bulk solid weight, even when the bulk material is at rest (in contrast to Newtonian liquids). A method for calculating the pressure in the silo's vertical section was derived from Janssen as early as 1895 [1]. The same method is applied in most currently silo standards, e.g., the European code [2], for calculating the silos loads for structural silo design.

<sup>1</sup> Prof. Dr. Dietmar Schulze, Dr. Dietmar Schulze GmbH, Wolfenbüttel, Germany.  
E-Mail: [mail@dietmar-schulze.de](mailto:mail@dietmar-schulze.de), Internet: [www.dietmar-schulze.de](http://www.dietmar-schulze.de)

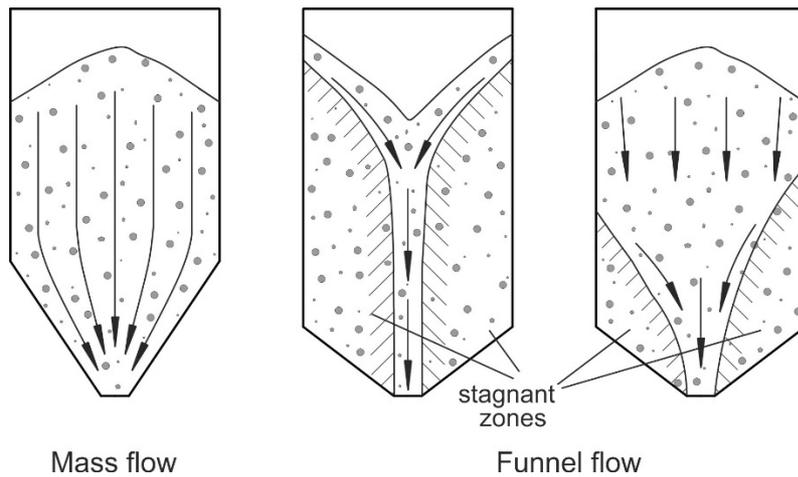


**Fig. 1:** a. Pressure in a silo filled with a fluid (imaginary); b. vertical stress after filling the silo with a bulk solid; c. vertical stress after the discharge of some bulk solid

The stresses acting in a hopper are different from those in the vertical section. Just after filling an empty silo, the so-called filling stress state (also: active stress state, Fig. 1b) prevails, where the vertical stress in the hopper decreases less in the upper part of the hopper and then more near the imaginary hopper apex. As soon as some bulk solid is discharged for the first time after filling, the stresses in the hopper change and the so-called emptying stress state (also: passive stress state) prevails, Fig. 1c. When flowing downwards in the hopper, the bulk solid is compressed in the horizontal direction so that the walls of the hopper carry a larger part of the weight of the bulk solid and, hence, the vertical stress in the lower part of the hopper is clearly smaller than after filling. In the emptying stress state, the vertical stresses in the lower part of the hopper are nearly proportional to the distance to the imaginary hopper tip or, in other words, the stresses are proportional to the local hopper diameter. This linear course of stress is called the radial stress field [3]. In principle, in the vertical section of the silo the stresses remain unchanged at discharge.

## 2 Flow profiles: Mass flow and funnel flow

Two different flow patterns can be observed if a bulk solid is discharged from a silo: Mass flow and funnel flow (Fig. 2). In case of mass flow, every particle of the bulk solid in the silo is moving during discharge. Mass flow is only possible, if the hopper walls are sufficiently steep and/or smooth, and the bulk solid is discharged across the whole outlet opening. If a hopper wall is too flat or too rough, funnel flow will prevail. In case of funnel flow, only the bulk solid in a channel above the opening flows downwards. The bulk solid adjacent to the hopper walls remains at rest and is called “dead” or “stagnant” zone. This bulk solid can be discharged only when the silo is emptied completely. The stagnant zones can reach the top level of the filling so that a flow funnel is formed at the surface. It is also possible that the stagnant zones exist only in the lower part of the silo so that funnel flow cannot be recognized by observing the surface of the filling.

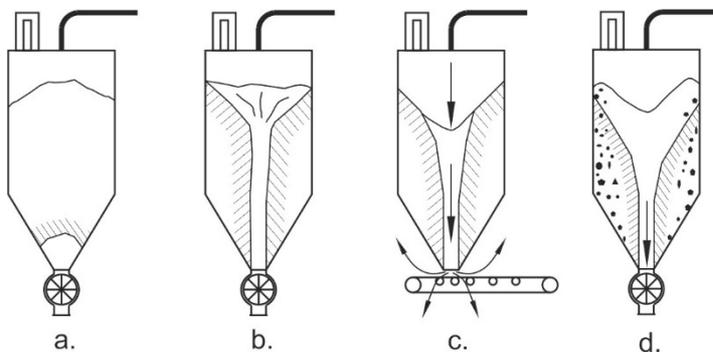


**Fig. 2:** Mass flow and funnel flow

### 3 Flow problems

Typical problems that arise when storing bulk materials in silos are:

- **Arching** occurs if a stable arch is formed above the outlet so that the flow of the bulk solid is stopped (Fig. 3a). In case of fine-grained, cohesive bulk solids, the reason of arching is the strength (unconfined yield strength) of the bulk solid which is caused by the adhesion forces acting between the particles. In case of coarse-grained bulk solid, arching is caused by interlocking and wedging of particles. Arching can be prevented by sufficiently large outlets.



**Fig. 3:** a. Arching; b. Ratholing; c. Irregular flow; d. Segregation

- **Ratholing** (Fig. 3b) occurs in case of funnel flow if only the bulk solid above the outlet is flowing out, and the remaining bulk solid - the stagnant zones - is consolidated and forms the rathole. The reason for this is the strength (unconfined yield strength) of the bulk solid. If the bulk solid consolidates increasingly with increasing time of storage at rest, the risk of ratholing increases. If a funnel flow silo is not emptied completely in sufficiently small regular time intervals, the period of storage at rest can become very large thus causing a strong time consolidation.
- **Irregular flow** (Fig. 3c) occurs if arches and ratholes are formed and collapse alternately. Thereby fine-grained bulk solids can become fluidized when falling downwards to the outlet opening, and flow out of the silo like a liquid. This behavior is called flooding. Flooding can cause dust, and a continuous discharge becomes impossible. In funnel flow silos, flooding

can also occur as a result of insufficient deaeration due to the short residence time in the flow zone when the bulk solid is discharged during filling.

- **Wide residence time distribution:** If stagnant zones are formed (funnel flow), the bulk solid in this zones is discharged only at complete emptying of the silo, whereas bulk solid, which is filled in later, but located closer to the axis of the silo, is discharged earlier. Because of that, a wide distribution of residence time appears which is disadvantageous in some cases (e.g. in case of storage of food or other products changing their properties with time).
- **Segregation:** If a heap is formed on the bulk solids' surface at filling of the silo, segregation is possible according to particle size or particle density (Figure 3c). In case of centric filling as shown in Figure 3c, the larger particles accumulate close to the silo walls, while the smaller particles collect in the center. In case of funnel flow, the finer particles, which are placed close to the center, are discharged first while the coarser particles are discharged at the end. If such a silo is used, for example, as a buffer for a packing machine, this behavior will yield to different particle size distributions in each packing. In case of a mass flow, the bulk solid will segregate at filling in the same manner, but it will become "remixed" when flowing downwards in the hopper. Therewith, at mass flow the segregation effect described above is reduced significantly.

In a funnel flow silo, all problems mentioned above can occur, while in case of mass flow only arching has to be considered. Segregation, ratholing, irregular flow and flooding of the bulk solid do not appear in a well-designed mass flow silo. The residence time distribution of a mass flow silo is narrow, because it acts as a „first in - first out" system.

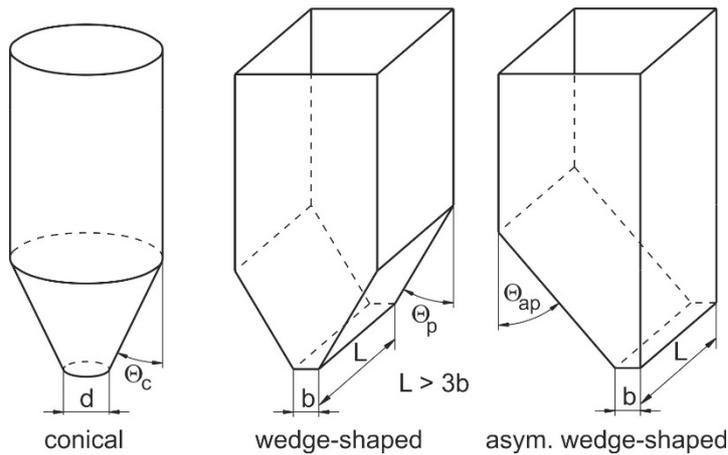
Two steps are necessary to design a mass flow silo: (1) The calculation of the required hopper slope which ensures mass flow, and (2) the determination of the minimum size of the outlet opening to prevent arching.

## 4 Silo design for flow

The flow behavior of a bulk solid is defined by several well-defined parameters [3–6]. In general, these are the bulk density,  $\rho_b$ , the effective angle of internal friction,  $\varphi_e$  (a measure for the internal friction of the bulk solid at steady-state flow), the unconfined yield strength,  $\sigma_c$ , and the wall friction angle,  $\varphi_x$ . For mass flow design, the wall friction angle  $\varphi_x$  is the most important parameter, whereby the unconfined yield strength,  $\sigma_c$ , is the most important parameter regarding arching. The wall friction angle,  $\varphi_x$ , is defined as the friction angle between the surface of the silo wall and the bulk solid. The unconfined yield strength,  $\sigma_c$ , is the compressive strength of a bulk solid. It must be considered that all these parameters are dependent on the stress level which is represented by the consolidation stress,  $\sigma_1$  [3–6].

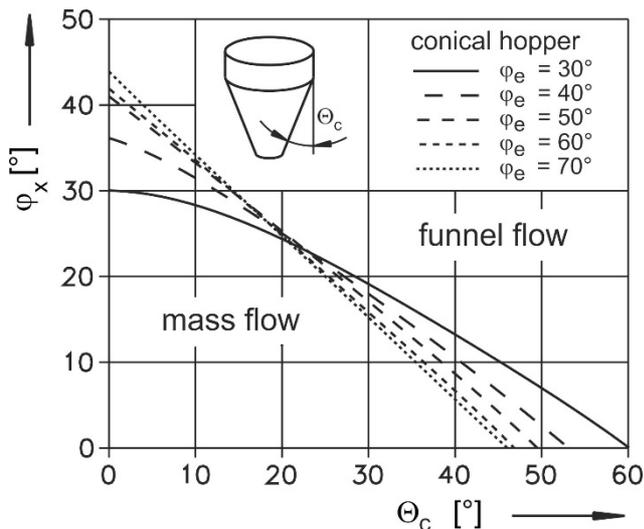
The parameters mentioned are measured in dependency on the consolidation stress with shear testers [3–6], e.g. with the Jenike shear tester or a Ring Shear Tester. The hopper slope required for mass flow and the minimum outlet size to prevent arching can be calculated with the measured values using Jenikes' theory [3]. This method showed its validity in many cases since the 1960's.

The basic geometrical shapes investigated by Jenike [3] are the conical hopper and the wedge-shaped hopper (Fig. 4). In addition, asymmetrical wedge-shaped hoppers are treated. However, an asymmetric wedge-shaped hopper has no advantages over the symmetrical wedge-shaped hopper with regard to the flow of bulk solid or best use of space. For all wedge-shaped hoppers it is assumed that the outlet slot length,  $L$ , is at least three times as large as the outlet slot width,  $b$  ( $L > 3b$ ). The goal of the design is to determine the necessary inclination of the hopper wall for mass flow and to determine the outlet size in such a way that no flow problems arise due to the formation of arches or ratholes.



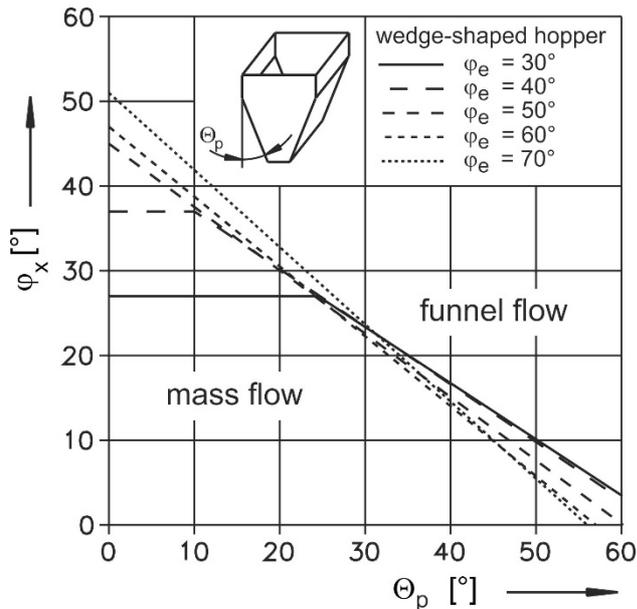
**Fig. 4:** Basic silo shapes: a. conical; b. wedge-shaped; c. asymmetric wedge-shaped

The borders between funnel and mass flow, which result from Jenike's calculations [3], are shown in Figure 5a for the conical hopper and in Figure 5b for the wedge-shaped hopper. In the diagrams the wall friction angle,  $\varphi_x$ , is plotted vs. the hopper wall angle,  $\Theta$ , measured against the vertical. The effective angle of internal friction,  $\varphi_e$ , which is a measure of the internal friction of the bulk solid, is the parameter of the mass flow/funnel flow borderlines. The borderlines separate all pairs of values leading to mass flow from those leading to funnel flow.



**Fig. 5a:** Mass flow diagram (conical hopper)

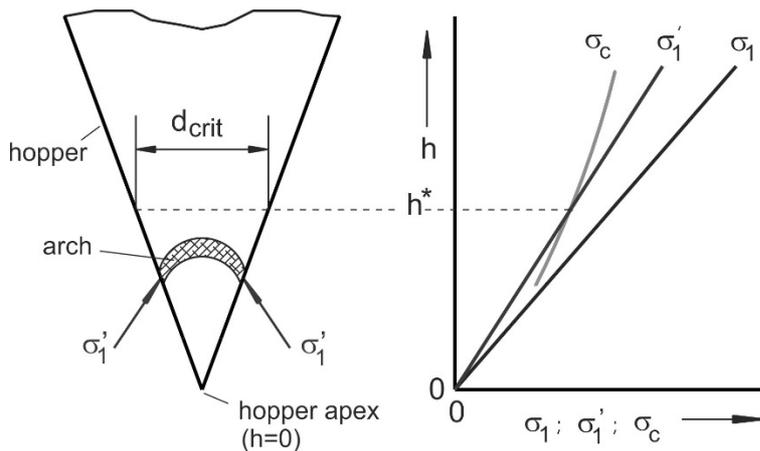
Conditions within the mass flow borderline yield mass flow whereas funnel flow is present in case of conditions outside of the borderline. If the wall friction angle,  $\varphi_x$ , and the effective angle of internal friction,  $\varphi_e$ , are known (based on shear tests), the maximum inclination angle,  $\Theta$ , of the hopper wall against the vertical which ensures mass flow can be determined with this diagram. The shape of the borderlines indicates that the larger the wall friction angle,  $\varphi_x$ , the steeper the hopper wall (smaller  $\Theta$ ) to achieve mass flow. The wedge-shaped hopper allows a somewhat larger wall inclination angle  $\Theta$  against the vertical with the same material properties (often  $8^\circ$  to  $10^\circ$  larger). This means that the walls of a wedge-shaped mass flow hopper can be flatter than the walls of a conical mass flow hopper [3, 5].



**Fig. 5b:** Mass flow diagram (wedge-shaped hopper)

When bulk solid flows in mass flow hopper, the radial stress field prevails in the lower part of the hopper as already described in Section 1 (see Fig. 1c). Thus, in the lower part of the hopper, the major principal stress,  $\sigma_1$ , is proportional to the local hopper diameter (Fig. 6).  $\sigma_1$  tends towards at the imaginary hopper apex. The major principal stress,  $\sigma_1$ , is acting as the consolidation stress thus determining the properties of the bulk solid, e.g. the bulk density,  $\rho_b$ , and the unconfined yield strength,  $\sigma_c$ .

The unconfined yield strength,  $\sigma_c$ , of a bulk solid can be measured for each major principal stress (consolidation stress),  $\sigma_1$  ([6]). The function  $\sigma_c = f(\sigma_1)$  is called the (instantaneous) flow function (Fig. 7). Usually, the unconfined yield strength increases with the consolidation stress. If the flow function has been measured, the unconfined yield strength,  $\sigma_c$ , can be plotted at each position of the hopper (Fig. 6).

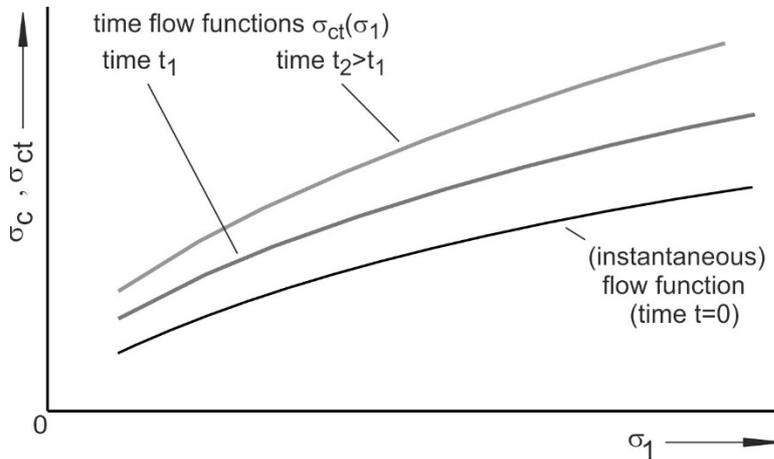


**Fig. 6:** Stress conditions in the hopper (emptying)

If a cohesive arch has formed in a hopper (Fig. 6), a force resulting from the weight of the bulk solid is transferred to the hopper walls. This results in the major stress,  $\sigma_1'$ , required to support a stable arch, like a bearing stress of a road bridge.  $\sigma_1'$  is proportional to the local hopper diameter such as  $\sigma_1$ .

A stable arch is only possible if the unconfined yield strength,  $\sigma_c$ , is larger than the stress,  $\sigma_1'$ , acting in the arch. This is the case beneath the point of intersection of the  $\sigma_c$  curve with the

$\sigma_1'$  line. Above the point of intersection, the unconfined yield strength is smaller than the major stress in the arch. In this case, the unconfined yield strength is not large enough to support an arch, i.e. an arch would not be stable at this position. The point of intersection defines that position in the hopper (height  $h^*$ , Fig. 6) where the hopper diameter is equal to the so-called critical diameter,  $d_{crit}$ , which must be exceeded if arching is to be avoided. If a smaller outlet opening would be chosen, flow promoting devices must be installed between the outlet opening and  $h^*$  [5].



**Fig. 7:** Flow function and time flow function

Some bulk solids tend to consolidate with time when stored at rest (time consolidation [5, 6]). It can be found a time flow function  $\sigma_{ct} = f(\sigma_1)$  (Fig. 7) for each storage time analogously to the instantaneous flow function. If the time flow function would be plotted in Fig. 6, a point of intersection of  $\sigma_1'$  and  $\sigma_{ct}$  would result located further upwards, i.e., at  $h > h^*$ . This means that larger outlets are required to prevent arching with increasing storage time at rest.

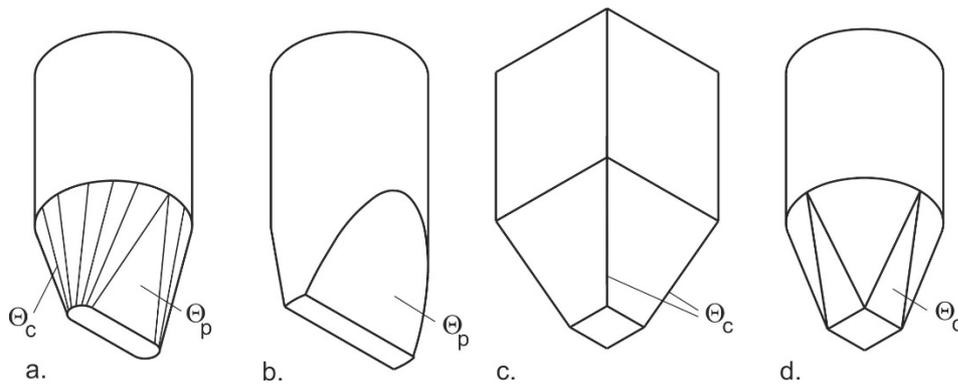
For the practical design of silos Jenike has derived diagrams and equations with which  $\sigma_1$  and  $\sigma_1'$  are determined in dependence on the flow properties of the bulk solid ( $\varphi_e$ ,  $\varphi_x$ ,  $\rho_b$ ) and the hopper shape ( $\Theta$ ). Finally, based on the measured instantaneous flow function (and time flow functions), the minimum dimensions of the outlet openings are determined for the basic hopper shapes shown in Fig. 4. A similar procedure is also available for the calculation of outlet dimensions to avoid the formation of ratholes in the case of funnel flow [3].

Examples of the design procedure are shown in [3, 5]. Depending on the stress dependence of the flow properties, the design may require iterations and thus a certain amount of time. Suitable programs, e.g. the program CAHD - Computer-aided hopper design [7] created by the author, can help here.

## 5 Selection of the hopper shape

Jenike's calculations (see layout diagrams, Fig. 5) relate to the basic shapes “conical hopper” and “wedge-shaped hopper”, and there are also some results for asymmetrical wedge-shaped hoppers (Fig. 4). For these shapes, the maximum wall inclination angle  $\Theta$  to achieve mass flow ( $\Theta_c$  for the conical,  $\Theta_p$  and  $\Theta_{ap}$  for the wedge-shaped hopper) and the outlet dimensions  $d$  and  $b$  to avoid arching can be determined. In case of the wedge-shaped hopper it is assumed that the influence of the vertical end walls can be neglected if the length of the outlet,  $L$ , is at least three times its width,  $b$ .

The results can also be applied to other hopper shapes in which elements of the basic shapes can be found. Some possibilities for the design of a mass flow silo are shown in Fig. 8 [3, 5, 8].



**Fig. 8:** Hopper shapes [9]

Variants a and b are just as favorable as a wedge-shaped hopper for achieving mass flow if the maximum angles of inclination indicated in Fig. 8 are not exceeded. The pyramid-shaped hopper (c) is unfavorable because here the bulk solid would have to flow from along the corners of the hopper walls towards the outlet opening. In doing so, friction on both adjacent walls would have to be overcome. This is difficult, i.e. it promotes the formation of stagnant zones in these areas. In order to achieve mass flow in such a hopper, the corners would have to be sufficiently rounded on the inside and their inclination to the vertical should be a maximum of  $\Theta_c$ . Since the corners between two walls of a pyramid-shaped hopper are always flatter than the walls itself, a pyramid-shaped hopper designed for mass flow would always be steeper than a corresponding conical hopper. Therefore, this hopper shape is generally unfavorable for achieving mass flow.

Variant d shows a transition from a cylindrical vertical section to a square outlet. Here the inclination of the hopper walls is to be determined in such a way that the maximum inclination to the vertical,  $\Theta_c$ , is not exceeded at any point.

A comparison of the wall inclination angles of the individual shapes shows that the variants in Fig. 8c/d have to be made the steepest in order to achieve mass flow. The conical hopper (Fig. 4) can be made somewhat flatter, and the flattest wall inclinations result from the geometries in Fig. 8a/b and the wedge-shaped hopper in Fig. 4.

Sometimes asymmetrical hoppers are favored (e.g. pyramid-shaped hoppers with four differently inclined walls). From the point of view of mass flow design, nothing speaks in favor of such a geometry. The symmetrical hopper requires the lowest height under the condition of mass flow [9]. Asymmetrical hoppers should therefore only be used if there is no other option for reasons of space.

## 6 Application of test results and silo design

In Section 4 the silo design procedure due to the theory of Jenike was described in a shortened way. Further details and information can be given besides the determination of the hopper slope for mass flow and the size of the outlet to prevent arching. Some examples are listed shortly in the following (further examples of silo design: [10–13]):

- Details of the hopper inclination angle and outlet dimension for different hopper shapes (see Fig. 8) and wall materials. This makes it possible to compare the manufacturing costs for different hopper shapes and materials [11, 14]. For example, it can be stated whether lining the silo walls (e.g. with cold-rolled stainless steel sheet) is a cost-effective alternative for a specific task.
- If the silo design results in a very steep mass flow hopper, or if in the case of a retrofit of an existing silo mass flow should be achieved without modifying the (too shallow) hopper

walls, inserts may be a solution. The inserts have to be designed based on the measured flow properties and Jenike's theory in order to achieve mass flow, and to avoid flow obstructions [15, 16].

- In the case of varying material properties (e.g. varying moisture contents [9, 17]), a statement can be made as to which state leads to the most unfavorable flow properties. If the silo is designed for the most unfavorable of the possible conditions, the function is always given.
- In the case of bulk solids with the tendency to time consolidation, it is possible to make a quantitative statement on the required outlet dimensions as a function of storage time. For example, a mass flow silo offers the possibility of moving the entire bulk solid in the silo by regularly discharging and re-filling a small amount of the stored bulk solid (recirculation). This completely reduces the effect of the time consolidation built up so far. With this measure, the time consolidation of the bulk solid and, thus, the required outlet size can be limited [11].
- Based on the measurement of the flow properties, the influence of additives such as flow agents can be determined in order to find the optimal composition [11, 18].
- When storing sensitive bulk solids, where there is a risk of attrition or product change due to the stresses in the silo, the measurement of the flow properties can be used to examine the stress above which this risk exists. Based on this data, a silo can be designed in such a way that only stresses occur that do not have a negative impact on product quality [12, 13].
- In order to prevent shocks or vibrations during the discharge of bulk solid from silos, special inserts (e.g. discharge tubes) can be dimensioned due to the properties of the bulk solid [11, 19].
- Shear tests can also be used for quality control or comparative tests [18].

## 7 Summary

Silo design for flow, like the design of other process components, is possible on the basis of material properties and calculation methods. Since poorly dimensioned silos can lead to operational disruptions and a reduction in product quality, the silo geometry must always be determined on the basis of the material properties. The effort required to determine the properties of the bulk solid for silo design for flow is low compared to the costs that arise from loss of quality, standstill, retrofitting or reconstruction of the silo.

## 8 References

- [1] Janssen, H.A.: Getreidedruck in Silozellen, Ztg. Ver. dt. Ing. 39 (1895), pp. 1045-1049
- [2] DIN EN 1991-4:2010-12 (2010) Eurocode 1: Einwirkungen auf Tragwerke – Teil 4: Einwirkungen auf Silos und Flüssigkeitsbehälter; Deutsche Fassung EN 1991-4:2006
- [3] Jenike, A.W.: Storage and Flow of Solids, Bull. No. 123, revised edition, Engng. Exp. Station, Univ. Utah, Salt Lake City (1970)
- [4] Schwedes, J.: Fließverhalten von Schüttgütern in Bunkern, Verlag Chemie, Weinheim (1968)
- [5] Schulze, D.: Powders and Bulk Solids – Behavior, Characterization, Storage and Flow, 2<sup>nd</sup> Ed., Springer (2021)
- [6] Schulze, D.: Flow Properties of Powders and Bulk Solids, <https://www.dietmar-schulze.de/pdf/flowproperties.pdf>
- [7] CAHD Computer-aided hopper design, Programm zur verfahrenstechnischen Siloauslegung, [www.dietmar-schulze.de](http://www.dietmar-schulze.de)
- [8] Woodcock, C.R.; Mason, J.S.: Bulk Solids Handling, Leonard Hill, Glasgow and London (1987)
- [9] Schulze, D., Schwedes, J.: Storage and Flow of Bulk Solids in Silos and Information for Planning new Installations, VGB-Kraftwerkst. (English Issue) 70 (1990) 9, pp. 665-669
- [10] Schulze, D.: Feeders and Flow Promoting Devices, in: Silos - Fundamentals of Theory, Behaviour and Design, edited by C.J. Brown and J. Nielsen, publ. by E & FN Spon, London and New York (1998), pp. 200–220 (ISBN 0-419-21580-8)
- [11] Schulze, D., Schwedes, J.: Examples of Modern Silo Design, Bulk Solids Handling 11 (1991) 1, pp. 47–52
- [12] Schulze, D.; Schwedes, J.; Leonhardt, C.; Kossert, J.: Schüttguttechnische Auslegung eines Silos zur Lagerung von 10.000 t Schwefel, Schüttgut 3 (1997) 3, pp. 299–305
- [13] Kwade, A.; Schulze, D.: Proper Silo Design for Food Products - Today Strategies for Avoiding Segregation, Degradation and Hang-ups, PARTEC 98, Preprints „1st Europ. Symp. Process Technology in Pharmaceutical and Nutritional Sciences“, 10.-12.3.98, Nürnberg, pp. 157–166
- [14] Schäfer, R.; Schröer, H.; Schwedes, J.: Auslegung einer Großsilo-Anlage für die Einlagerung eines feuchten Schüttgutes und praktische Erfahrungen. Zement Kalk Gips 75 (1986) 11, pp. 587–592
- [15] Schulze, D.: Möglichkeiten der Silogestaltung, Schüttgut 1(1995) 1, pp. 19–25
- [16] Johanson, J.R.: Controlling flow patterns in bins by use of an insert. Bulk Solids Handling 2 (1983) 3, pp. 495–498
- [17] Schulze, D.; Schwedes, J.: Das Fließverhalten und die Silolagerung von REA-Gips, Chem.-Ing.-Techn. 63 (1991) 3, pp. 256–257
- [18] Schulze, D.: Vergleich des Fließverhaltens leicht fließender Schüttgüter, Schüttgut 2 (1996) 3, pp. 347–356
- [19] Schulze, D.; Schwedes, J.: Tests on the Application of Discharge Tubes, Bulk Solids Handling 12 (1992) 1, pp. 33–39